Plane-Change Requirements Associated with Rendezvous in a Lunar Satellite Orbit

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Plane-change requirements imposed by time-varying Earth-moon spacecraft-orbit geometry as a consequence of the desire to rendezvous in a lunar satellite orbit during a round-trip lunar mission are discussed Equations are developed which, for parameters of orbit inclination about the moon, lunar-surface launch latitude, and staytime on the lunar surface, define the plane change required to rendezvous A method of minimizing the maximum plane change needed to rendezvous during the stay is presented The angle of intersection between the lunar departure asymptote and the spacecraft satellite orbit is defined in terms of characteristics of the outbound and return trajectories, orbit inclinations, and staytimes This angle represents the minimum plane change needed to return to Earth Characteristics of lunar-centered hyperbolic trajectories needed to develop the equations of the departure plane-change requirement are established by means of three-dimensional joined conic analyses of Earth-moon trajectories Applications of analyses similar to those given in this paper (in some cases, actual numerical results) to interplanetary missions where rendezvous in orbit is employed are indicated

Nomenclature

- **a** = unit vector in direction of arrival asymptote
- **b** = unit vector in direction of departure asymptote
- **i**,**j** **k** = unit vectors along coordinate axes
- L = Earth launch point
- M = moon's position at arrival
- **N** = unit vector normal to spacecraft orbit plane
- r = radial distance from focus of conic
- T = elapsed time since moon passed through ascending node (with respect to earth equatorial plane)
- t = staytime on lunar surface
- V = velocity
- V_H = hyperbolic excess velocity
- α = true angle between V_H and moon's orbital velocity vector
- β = angle of intersection between V_H and moon's orbit plane
- Γ = inclination of lunar satellite orbit
- γ = flight-path angle (positive up from local horizontal)
- Δ = angle of intersection between satellite orbit plane and departure asymptote
- δ = declination of moon (positive north)
- η = angle between moon's velocity vector and projection of V_H in lunar orbit plane
- θ = geocentric trajectory angle
- ι = lunar orbit inclination with respect to Earth's equator
- κ = true anomoly of descent point on a satellite orbit, measured from ascending node with respect to lunar equator
- λ = latitude on the lunar surface (positive north)
- μ = gravitational constant
- σ = azimuth, degree east of north
- τ = out-of plane distance between lunar satellite orbit and lunar surface departure point
- υ = angle between geocentric trajectory plane and lunar-orbit plane
- ϕ = Earth latitude (positive north)
- ω = lunar rotation rate about earth (also equals rate about polar axis)

Subscripts

- B = boost
- C = coast
- m = refers to moon in Earth equator reference system
- P = parking orbit
- R = re entry
- 0 = departure from Earth
- 1 = on geocentric trajectory at distance $r = r_m$
- 2 = return to Earth

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1 Introduction

ONE method of carrying out a manned round-trip lunar mission involves injection of a spacecraft into a lunar satellite orbit and the subsequent descent to the lunar surface of a landing vehicle The return trip to Earth would then require that the landing vehicle perform an on-orbit rendezvous with the spacecraft This paper discusses plane-change maneuvers that are required to rendezvous and then to depart from the satellite orbit These requirements depend upon the directions of the asymptotes of the lunar approach and departure hyperbolas, staytime at the moon, inclination of the spacecraft rendezvous orbit, and the latitude of the landing site

Consider first the trajectory from lunar liftoff to rendezvous As the moon rotates about its polar axis, the out-of-plane distance from the launch site to the orbit plane varies and, for inertially stable orbits, is cyclic with a period equal to the moon's sidereal period (about 27 days) This distance represents the minimum angle of intersection between the spacecraft orbit plane and the landing-vehicle ascent-trajectory plane A combination of landing-site latitude and spacecraft-orbit inclination makes return at a nominal time possible without a plane change However, if the nominal staytime is a sizeable percentage of the sidereal period, and liftoff must occur earlier or later than planned, large plane changes, as high as 90°, may be necessary

After rendezvous has been accomplished, a second plane-change maneuver is required of the spacecraft to depart from orbit An approach hyperbola with the resulting spacecraft orbit inclination can always be found which contains both the approach asymptote and the departure asymptote at some particular time without recourse to a plane-change maneuver Since the departure asymptote direction changes with time, however, the asymptote, in general, will not lie in the spacecraft orbit plane if departure cannot occur when planned, even in the limiting case of arrival followed by an immediate unscheduled departure

The amount of rotation of the departure asymptote is dependent upon the angular distance swept out by the moon in its orbit about Earth For lunar missions, as shown in Fig 1, the departure asymptote will rotate 360° relative to the spacecraft orbit every lunar month Arrival occurs when the moon is located at point 1 in its orbit about Earth The direction of the approach asymptote, defined by the unit vector **a** in the direction of the approach asymptote, is η_a For simplicity (in the figure only), assume that the approach and departure asymptotes lie in the moon's orbit plane and that the spacecraft orbit plane about the moon is normal to the lunar orbit plane Now, as the moon rotates about Earth, the direction η_b of the unit vector **b** in the direction of the departure asymptote remains constant with respect to the Earth-moon line If perturbations are ignored, the spacecraft orbit plane maintains a constant inertial orientation The departure asymptote, therefore, intersects the spacecraft orbit plane at angle Δ, which depends upon the staytime (to be exact, Δ depends upon the angular distance traveled by the moon during the staytime) The angle Δ is the minimum plane-change maneuver that would be required by the spacecraft at departure Even for short-duration stopover missions, which exhibit only minor rotations of the departure asymptote, a spacecraft departure plane-change could be required if the orbit inclination were chosen merely to minimize the first plane-change requirement, namely, that needed to perform rendezvous

One of the more significant conclusions reached in the following discussions is that the only parameter that exerts much influence on the departure plane-change requirement for a given staytime is the inclination of the spacecraft orbit around the moon Merely allowing the inclination of the geocentric return trajectory to vary has a negligible effect on the requirements

To develop the equations of the two plane-change maneuvers (i e , rendezvous and departure), only characteristics of the Earth-moon trajectories in the vicinity of the moon are of importance; results of precise trajectory computations are unnecessary Consequently, the method of joined conics is used to define the lunar-arrival and departure geometric characteristics Explicit relationships among the variables, therefore, can be readily developed

2 Three-Dimensional Joined Conics

This section discusses the method of joined conics used to determine the lunar arrival and departure conditions The results obtained are necessary to ascertain the plane change required of the spacecraft in order to enter the lunar departure trajectory discussed in Sec 4 Some additional discussion of joined-conic Earth-moon trajectories is available (see, for example, Refs 1 and 2)

To determine the geometric relationships between the outbound (return) trajectory and the moon, it is necessary to consider first the Earth-departure (arrival) conditions The relationships can be derived with the aid of Fig 2, which shows the intersections of the lunar orbit plane and the outbound trajectory plane with a nonrotating terrestrial sphere The assumption is made that the powered flight path, the parking orbit (if any), and the coast trajectory all lie in one plane

The required launch azimuth σ_0 is given by

$$\cos\sigma_0 = \frac{\sin\delta - \sin\phi_0 \cos(\theta_B + \theta_P + \theta_C)}{\cos\phi_0 \sin(\theta_B + \theta_P + \theta_C)} \qquad (2\ 1)$$

The angle υ between the lunar orbit plane and the geocentric trajectory plane is merely

$$\upsilon = \sigma_1 - \sigma_m \qquad (2\ 2)$$

where

$$\sin\sigma_1 = \cos\phi_0 \sin\sigma_0/\cos\delta$$

and

$$\sin\sigma_m = \cos\iota_m/\cos\delta$$

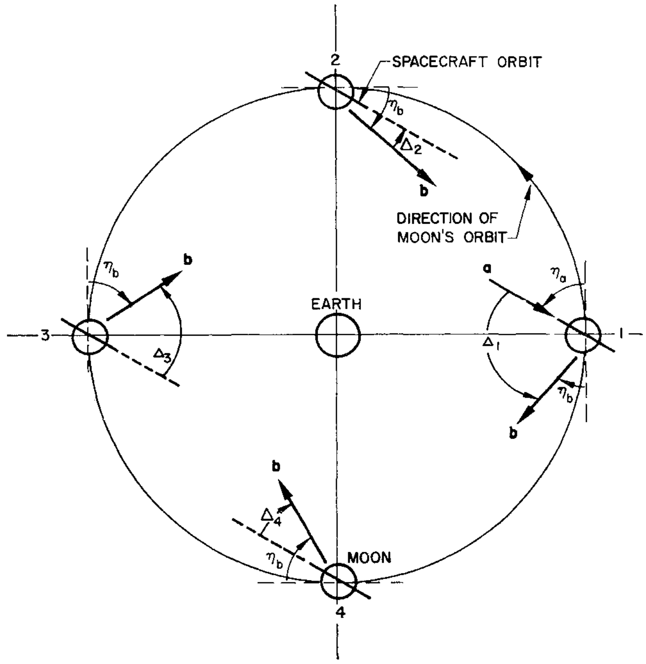

Fig 1 Effect of staytime on departure plane-change requirement

The analogous expression for arrival azimuth of the return trajectory is

$$\cos\sigma_2 = \frac{\sin\phi_2 \cos(\theta_C + \theta_R) - \sin\delta}{\cos\phi_2 \sin(\theta_C + \theta_R)} \qquad (2\ 3)$$

Equation (2 3) assumes that the Earth re-entry trajectory is in the same plane as the return coast trajectory Similarly,

$$\upsilon = \sin^{-1}(\cos\phi_2 \sin\sigma_2/\cos\delta) - \sin^{-1}(\cos\iota_m/\cos\delta) \qquad (2\ 4)$$

Having obtained the expression for υ, the velocity requirements at the moon can be determined by the method of joined conics It is assumed that a geocentric trajectory is followed to the radial distance of a massless moon (r_m = 207,000 naut miles) and that the line of nodes between the lunar orbit plane and the geocentric trajectory plane lies along the Earth-moon line Trajectory parameters at this point are then transformed into lunar-referenced parameters at an "infinite" distance from the moon Properties of the geocentric trajectory are shown in Fig 3 The velocity relative to the Earth at the distance r_m is found from the

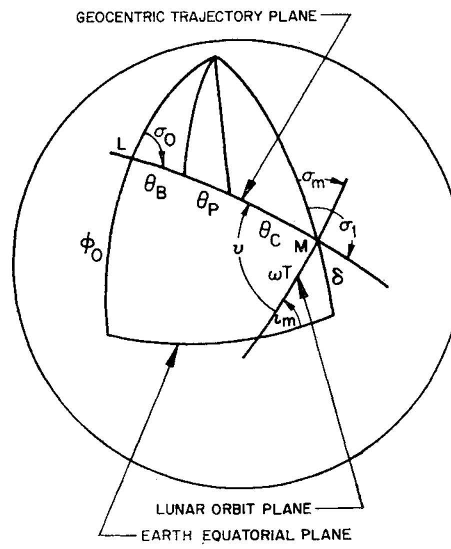

Fig 2 Geometry of Earth-centered motion

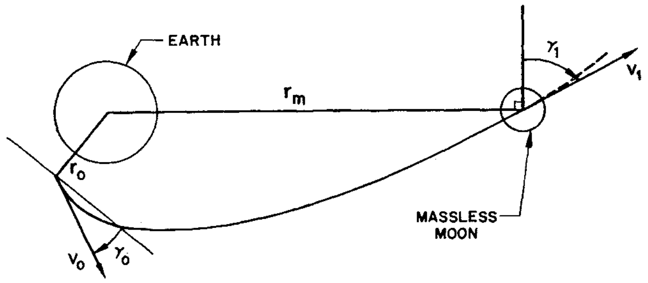

Fig 3 Joined conic trajectory

energy equation:

$$V_1^2 = V_0^2 + 2\mu[(1/r_m) - (1/r_0)] \tag{2 5}$$

where $\mu = 1\,4077 \times 10^{16}$ ft³/sec² for the Earth The minimum possible V_1 occurs when apogee radius equals the lunar orbital radius, so that

$$V^2_{1(\min)} = 2\mu\left(\frac{1}{r_m} - \frac{1}{r_0 + r_m}\right) \tag{2 6}$$

For r_0 = 3590 naut miles, $V_{1(\min)}$ = 618 fps, which corresponds to a minimum Earth departure velocity of 35,607 fps

The final parameter required, found from conservation of angular momentum, is the angle γ_1:

$$\cos\gamma_1 = r_0V_0\cos\gamma_0/r_mV_1 \tag{2 7}$$

It is important to realize that, for one-way travel times between the wide limits of 1½ to 3½ days, the departure velocity varies only from about 36,275 fps to about 35,580 fps at r_0 = 3590 naut miles † Similarly, it is likely that r_0 will be confined to rather narrow limits, about 3520 to 3740 naut miles, and γ_0 will usually be near zero to reduce Earth ascent-vehicle gravity losses Similar statements can be made concerning return travel times and the narrow limits on vacuum perigee radii Thus it can be concluded that the tangential velocity $V_1 \cos\gamma_1$ is essentially constant, varying between about 610 and 625 fps The fact that $V_1 \cos\gamma_1$ is constant and is rather small will be of importance in discussing the plane change needed to depart from the lunar satellite orbit

The hyperbolic excess velocity at the moon may now be computed with the aid of Fig 4 In Fig 4, ABC is the lunar orbit plane, ADE is the plane of the geocentric trajectory, and ADB is a plane defined by the geocentric velocity vector and the hyperbolic excess velocity vector:

$$V_H{}^2 = V_1{}^2 + V_m{}^2 - 2V_1V_m\cos\gamma_1\cos\upsilon \tag{2 8}$$

The velocity relative to the moon at any radius from the moon's center can be found from energy considerations to be

$$V^2 = V_H{}^2 + (2\mu/r) \tag{2 9}$$

where μ for the moon = $1\,733 \times 10^{14}$ ft³/sec² From Fig 4

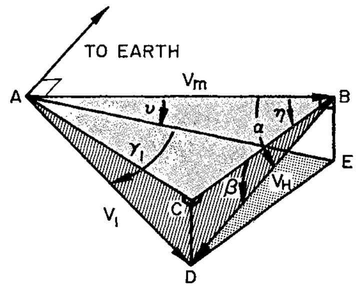

Fig 4 Lunar approach velocity diagram

† Travel times associated with these departure velocities were obtained digitally, not by a joined conic analysis

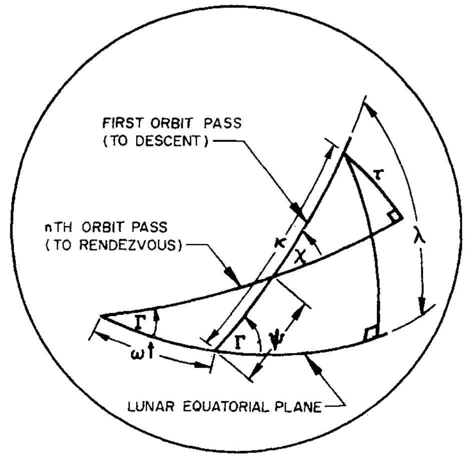

Fig 5 Rendezvous geometry (shown as orbit plane regressing about a stationary moon)

we also determine that

$$\cos\alpha = (V_H{}^2 + V_m{}^2 - V_1{}^2)/2V_mV_H \tag{2 10}$$

$$\cos\eta = \cos\alpha/\cos\beta \tag{2 11}$$

and

$$\sin\beta = V_1\sin\upsilon\cos\gamma_1/V_H \tag{2 12}$$

which, by using Eq (2 7) and assuming $\gamma_0 = 0$, becomes

$$\sin\beta = r_0V_0\sin\upsilon/r_mV_H \tag{2 13}$$

The angles α and β completely specify the direction of the approach asymptote Now, if small changes are made in the Earth departure conditions (e g , $\Delta V_0 \approx 50$ fps), the position of V_H (i e , the position of the approach asymptote) can be altered, but its direction remains essentially constant This means that, as discussed in Ref 3, if we define a unit vector **a**, which passes through the center of the moon parallel to the approach asymptote, all possible trajectory planes of this family relative to the moon can be thought of as being generated by rotating any particular plane about **a** It should be apparent that any inclination Γ can be obtained in this way subject only to the limitation that $\Gamma \geq \beta$ ‡ Note that, using the range of departure-velocity and radius values in equations developed this far, even in the limiting case of

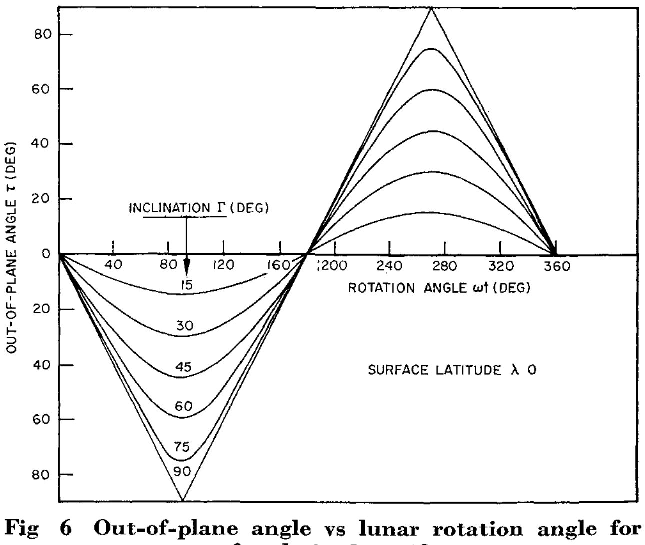

Fig 6 Out-of-plane angle vs lunar rotation angle for surface latitude = 0°

‡ It is assumed that the lunar orbit plane and the lunar equatorial plane are identical They actually differ by no more than 6 5°

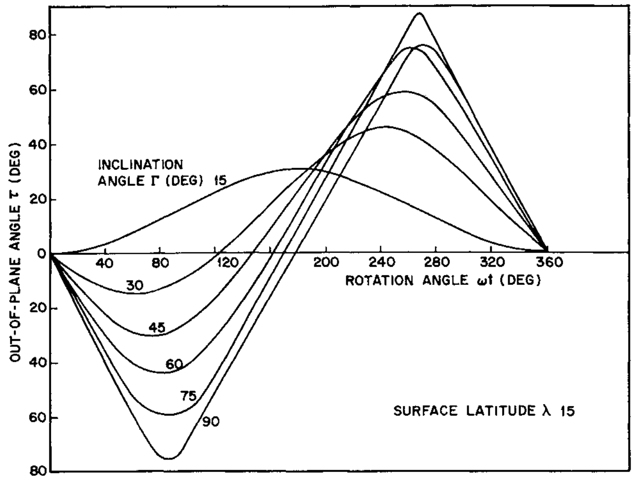

Fig 7 Out-of-plane angle vs lunar rotation angle for surface latitude = 15

$v = \pi/2$, β does not exceed about $10\frac{1}{2}°$ Also, it easily can be shown that for outbound trajectories that arrive in the vicinity of the moon on the first pass, $0 < \alpha < \pi/2$, and that α is a monotonically decreasing function of one-way travel time

The foregoing results have been derived for outbound trajectories They also apply to direct-return trajectories, however, if suitable algebraic signs are used In particular, for outbound trajectories, β is positive if the approach is from the north; for return trajectories, β is positive if the departure is toward the south Also, for outbound trajectories, α is measured from the positive V_m axis; for return trajectories, α is measured from the negative V_m axis In this connection, it is shown in Ref 4 that a direct moon-earth trajectory that has the same energy and geocentric inclination as the outbound trajectory is an image of the outbound trajectory; i e , $\beta_b = \beta_a$ and $\alpha_b = \alpha_a$

3 Plane Change Needed to Rendezvous

Consider a vehicle in a satellite orbit at inclination Γ with respect to the lunar equator (Fig 5) At some point retrothrust is employed, and a landing vehicle separates from the spacecraft Following a coplanar trajectory, the descent vehicle lands at latitude λ

At a later time the ascent vehicle will rise from the surface and must enter the plane of the orbiting spacecraft The problem is to determine the ascent vehicle out-of-plane maneuver requirements τ Functions of surface staytime, these

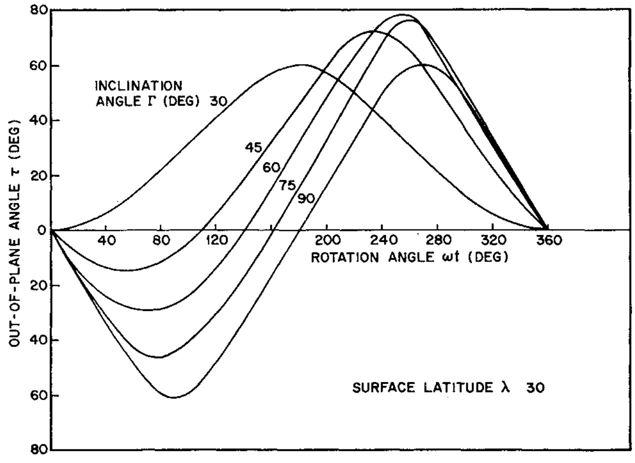

Fig 8 Out-of-plane angle vs lunar rotation angle for surface latitude = 30°

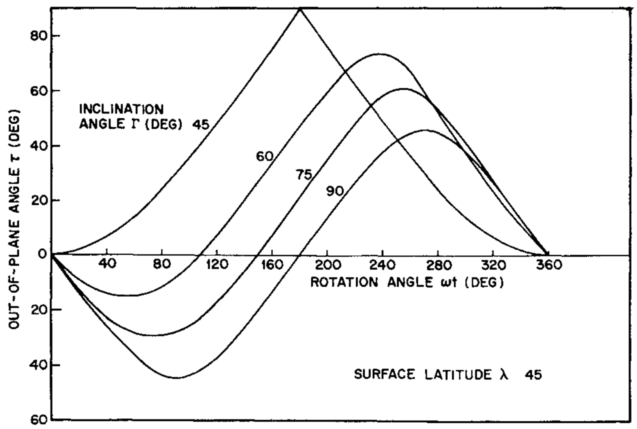

Fig 9 Out-of-plane angle vs lunar rotation angle for surface latitude = 45°

requirements can be ascertained as follows § The angle τ is first found to be

$$\tau = \sin^{-1}[\sin\chi \sin(\kappa - \psi)] \qquad \kappa \leq 90°$$
$$\tau = \sin^{-1}[\sin\chi \sin(\kappa + \psi)] \qquad \kappa > 90° \qquad (3\ 1)$$

where

$$\kappa = \sin^{-1}(\sin\lambda/\sin\Gamma)$$

and the auxiliary variables χ and ψ are

$$\chi = \cos^{-1}(\cos^2\Gamma + \sin^2\Gamma \cos\omega t)$$

and

$$\psi = \sin^{-1}(\sin\Gamma \sin\omega t/\sin\chi)$$

Simplification gives

$$\tau = \sin^{-1}[(1 - \cos\omega t) \sin\lambda \cos\Gamma - (\sin^2\Gamma - \sin^2\lambda)^{1/2} \sin\omega t] \qquad \kappa \leq 90°$$
$$(3\ 2)$$
$$\tau = \sin^{-1}[(1 - \cos\omega t) \sin\lambda \cos\Gamma + (\sin^2\Gamma - \sin^2\lambda)^{1/2} \sin\omega t] \qquad \kappa > 90°$$

where ωt is the angle through which the moon has rotated about its polar axis during the staytime t and ω = 13 15 deg/day In Figs 6–11, the angle τ is shown as a function of ωt for parameters of λ and Γ Note that touchdown at a given latitude from a given orbit can occur at two distinct longitudes depending upon whether the true angle of touchdown

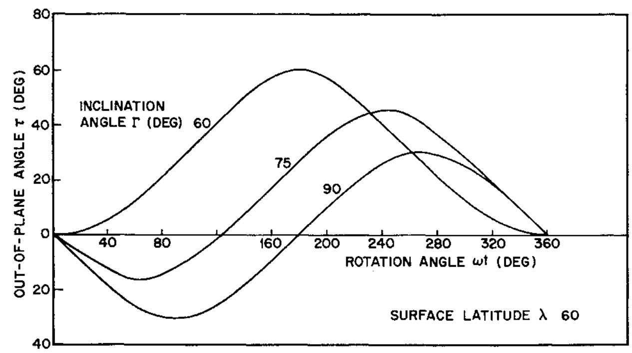

Fig 10 Out-of-plane angle vs lunar rotation angle for surface latitude = 60°

§ Because the nodal regression rate, caused by the oblateness of the moon, of a low altitude orbit of low eccentricity does not exceed 1°/day, no orbit perturbations are considered in the following analysis When necessary, this effect may be treated as an effective increase in the lunar rotation rate

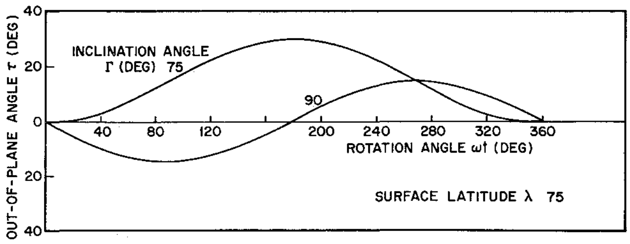

Fig 11 Out-of-plane angle vs lunar rotation angle for surface latitude = 75°

κ measured from the ascending node is greater or less than 90° As noted, the data of Figs 6–11 are for $\kappa < 90°$ If $\kappa > 90°$, each curve should be translated to the left so that the point at which it crosses the abscissa occurs at $\omega t = 0$ For retrograde orbits, the data shown apply to values of $\kappa > 90°$; the translated curves would apply to values of $\kappa < 90°$ For southerly latitudes, the curves apply to angles from the descending node of less than 90°

It is seen that for $\omega t \leq 180°$ (about 2 weeks' staytime), a combination of λ and Γ can be found that yields $\tau = 0$ at nominal departure time However, mission requirements may dictate that return capability exists throughout the entire stay This requirement can lead to a different criterion in the selection of inclination for a given landing latitude, namely, to choose Γ so that the maximum value of τ during the stay is minimized This value of τ can be described as a "minimax" τ The corresponding value of τ occurs when τ_1 (computed where $\partial\tau/\partial\omega t = 0$) equals the negative of τ_2 (computed at nominal ωt) as illustrated in Fig 12

Differentiation of Eq (3 2) shows that $\tau_1 = \lambda - \Gamma$ Using $\tau_2 = \Gamma - \lambda$ in Eq (3 2), a second relationship between λ and Γ is found Explicit relations exist only at $\omega t = 90°$ and $\omega t = 180°$ These relations are

$$\left.\begin{aligned} &\tan\lambda = \tfrac{4}{5}\tan\Gamma \\ &\tan\tau = -\tan\Gamma/(5 + 4\tan^2\Gamma) \end{aligned}\right\} \omega t = 90° \qquad \left.\begin{aligned} &\tan\lambda = \tfrac{1}{3}\tan\Gamma \\ &\tan\tau = -2\tan\Gamma/(3 + \tan^2\Gamma) \end{aligned}\right\} \omega t = 180° \tag{3 3}$$

For other values of $\omega t < 180°$, an iterative procedure must be used

For $\omega t > 180°$, the technique just described is not applicable since, in general, there is no combination of Γ and λ that yields $\tau = 0$ at nominal departure time From examination of Figs 6–11, however, it is seen that, depending on the value of λ, either $\lambda = \Gamma$ or $\Gamma = 90°$ yields the minimax τ:

$$\left.\begin{aligned} &\text{If } \Gamma = \lambda, \quad \text{then } \tau_1 = 2\lambda \\ &\text{If } \Gamma = 90°, \text{then } \tau_2 = 90 - \lambda \end{aligned}\right\} \tag{3 4}$$

At $\lambda = 30°$, $\tau_1 = \tau_2$ Consequently, we conclude that for $\lambda \leq 30°$, $\Gamma = \lambda$; and for $\lambda \geq 30°$, $\Gamma = 90°$ The minimax τ is shown in Fig 13 The interpretation is as

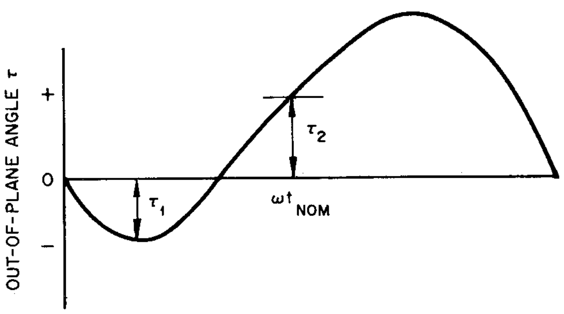

Fig 12 Illustration of minimax τ

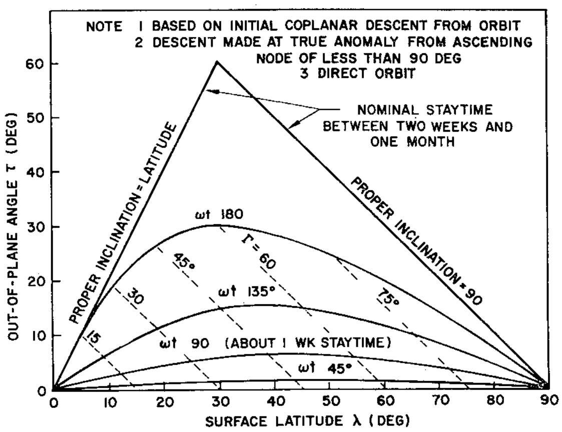

Fig 13 Minimum-maximum out-of-plane angle from landing site to spacecraft orbit plane

follows: for a given staytime (ωt = const), if the inclination shown associated with the given latitude is employed, the corresponding value of τ will never be exceeded during that entire staytime; the actual τ, depending upon the actual time of departure, may, of course, be less If the proper combinations are not used, the value of τ could be less, depending again upon the actual departure time, but will certainly be greater at some other departure time prior to nominal

It should be apparent that all results established in this section apply to any rotating body For rapidly rotating bodies such as Earth or Mars, however, the inteipretation of the numerical results is somewhat different Since the ascent-to-rendezvous requirements will be cyclic, with a period of about one day, the data can be used to analyze daily launch-window requirements For example, if ωt = 45°, the plane-change requirement on the moon would be that which would occur after a stay of 3 4 days; this same requirement would correspond to a 3-hr launch-window on Earth

4 Plane Change Required to Depart

In order to determine the plane change required of the spacecraft to depaıt from orbit, we first determine the equation of the unit normal **N** to the spacecraft orbit plane A coordinate system that is consistent with vectorial notation will be used to develop this equation (see Fig 14)

With reference to the previously defined orientation of the angles α and β, it is seen that

$$\beta_a' = 90 + \beta_a \qquad \alpha_a'(t) = 180 + \alpha_a(t) \tag{4 1}$$

One line in the orbit plane is the unit vectoı **a** in the direction of the arrival asymptote, rotated about the z axis through the angle ωt Its equation is

$$\mathbf{a} = \cos\alpha_a'(t)\mathbf{i} + [1 - \cos^2\alpha_a'(t) - \cos^2\beta_a']^{1/2}\mathbf{j} + \cos\beta_a'\mathbf{k} \tag{4 2}$$

where

$$\cos\alpha_a(t) = \cos\beta_a \cos(\eta_a - \omega t)$$

Since **a** lies in the orbit plane and **N** is a unit normal to the plane, **N a** = 0, where $\mathbf{N} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ Finally, we note that $z = \cos\Gamma$ and $x^2 + y^2 + z^2 = 1$ After some simplification, we make use of Eq (4 1) and apply the dot-product operation to obtain the components of **N**:

$$\left.\begin{aligned} x &= -\cos(\eta_a - \omega t)\cos\Gamma\tan\beta_a \pm \\ &\qquad (\sin^2\Gamma - \sin^2\beta_a)^{1/2}\sec\beta_a\sin(\eta_a - \omega t) \\ y &= -\sin(\eta_a - \omega t)\cos\Gamma\tan\beta_a \pm \\ &\qquad (\sin^2\Gamma - \sin^2\beta_a)^{1/2}\sec\beta_a\cos(\eta_a - \omega t) \\ z &= \cos\Gamma \end{aligned}\right\} \tag{4 3}$$

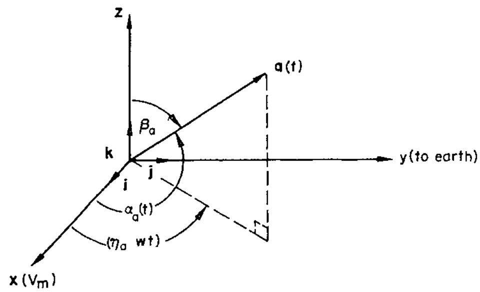

Fig 14 Lunar-centered coordinate system

If we denote the unit vector in the direction of the departure asymptote by **b**

$$\mathbf{b} = \cos\alpha_b'\mathbf{i} + (1 - \cos^2\alpha_b' - \cos^2\beta_b')^{1/2}\mathbf{j} + \cos\beta_b'\mathbf{k} \quad (4\ 4)$$

Again, using the original notation and keeping in mind the definition of the algebraic signs, **b** can be rewritten as

$$\mathbf{b} = -\cos\eta_b \cos\beta_b\mathbf{i} + \sin\eta_b \cos\beta_b\mathbf{j} - \sin\beta_b\mathbf{k} \quad (4\ 5)$$

Now, **N** **b** = $\cos\Delta'$, where Δ' is the acute angle between **N** and **b** Hence **b** intersects the spacecraft orbit plane at an angle $\Delta = (90 - \Delta')$

The final expression for Δ is as follows:

$$\sin\Delta = \cos\Gamma[\tan\beta_a \cos\beta_b \cos(\eta_a + \eta_b - \omega t) - \sin\beta_b] \pm \sec\beta_a \cos\beta_b(\sin^2\Gamma - \sin^2\beta_a)^{1/2} \sin(\eta_a + \eta_b - \omega t) \quad (4\ 6)$$

The physical meaning of the quadratic in the x component of the normal vector (and, hence, the double value of Δ) is that for any approach asymptote direction, two planes may be found with the proper inclination Γ However, unless **a** passes through the desired landing site, only one of these planes will pass over the site selected, so that for a particular mission the angle Δ is uniquely defined For direct orbits, if **a** lies within $\pm 90°$ of the descending node between the satellite orbit plane and the lunar equatorial plane, the plus sign in Eq (4 6) should be used; if **a** lies within $\pm 90°$ of the ascending node, the minus sign should be used If the spacecraft is in a retrograde orbit, opposite statements hold true

Even by treating the quantity $(\eta_a + \eta_b - \omega t)$ as an auxiliary variable, Δ still will depend on four parameters For this reason, generalized charts are rather difficult to present A representative case is shown in Fig 15 Assume, for this example, that $\eta_a = \eta_b = 50°$ and $\beta_a = \beta_b = 0°$ This corresponds to a 60-hr one-way trip, with the lunar orbit plane containing both the outbound and return trajectories Assume that all desired landing sites lie on the 0° meridian, i e , on the meridian of the Earth-moon line If no arrival plane change is made, there is a one-to-one correspondence between site latitude and orbit inclination Therefore, to illustrate the effects of latitude on the asymptote

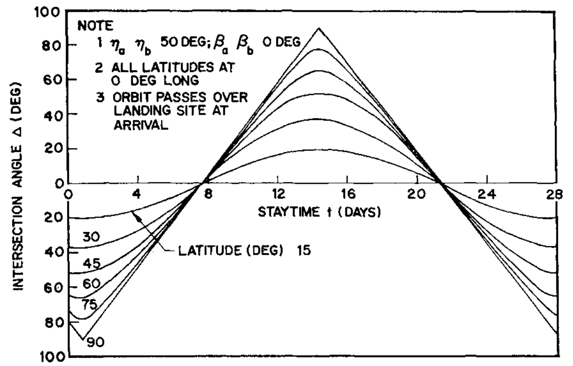

Fig 15 Angle of intersection between departure asymptote and spacecraft orbit plane

intersection angle Δ, Fig 15 shows latitude as a parameter It is seen that the departure plane-change requirements are almost at the maximum values at arrival but reduce to zero after about $7\frac{1}{2}$ days It is also observed that high latitudes dictate larger plane changes so that, for off-nominal returns, the requirements might well be excessive

It is important to note that, since β_a and β_b are rather restricted in value (since $V_1 \cos\gamma_1$ is small and constant) and since η_a and η_b are dependent almost entirely on travel time, the only parameter that can be used to mininize Δ is inclination Γ; changing the geocentric inclination is of little significance In other words, for a given travel time, merely changing the orientation of the geocentric trajectories will not materially influence the plane-change requirements

It should be pointed out that the analysis presented in this section is similar to that which can be employed in an analysis of interplanetary missions In the case of interplanetary missions the sun assumes the role of Earth, and the arrival planet (e g , Mars) assumes the role of the moon For interplanetary missions, however, the directions of the approach and departure asymptotes should be based on digitally computed trajectories Data for approach and departure asymptote directions can be found, for instance, in Ref 5

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